TONBRIDGE SCHOOL

Scholarship Examination 2004

MATHEMATICS II

Wednesday 5th May 2004
2.00 p.m.

Time allowed: 1 hour 30 minutes

Answer as many questions as you can.
All the questions carry equal marks.

All answers must be supported by adequate explanation.
Calculators may be used in any question.

1. The sum of the first n cube numbers, $1^3 + 2^3 + \ldots + n^3$, is given by the formula $\frac{1}{4} n^2(n+1)^2$.

Use this formula to answer the following questions: do <u>not</u> round-off your answers.

(a) What is the sum of the first 100 cube numbers?

(b) What is n if $1^3 + 2^3 + \ldots + n^3 = 44100$?

(c) What is $51^3 + 52^3 + \ldots + 99^3 + 100^3$?

(d) What is $2^3 + 4^3 + 6^3 + \ldots + 198^3 + 200^3$?

2. Three circular coins of different sizes are pushed together on a tabletop so that each just touches the other two. If the respective distances between the centres of the coins are 17 mm, 20 mm and 23 mm, find the radius of each coin.

(You may find it useful to draw a diagram and consider how the distance between the centres of two coins is related to their radii.)

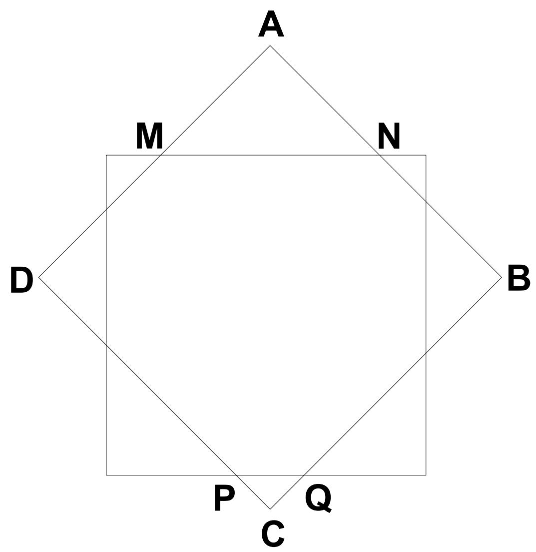
3. In this question, you may use the fact that the total surface area of a solid cylinder with radius r and length h is made up of its two ends (each of area πr^2) and its curved surface area ($2\pi rh$).

(a) Show that a cylinder with radius 10 cm and length 12 cm has a total surface area of approximately 1382.3 cm².

(b) A hole of radius 2 cm is drilled lengthwise through the cylinder in (a).
Find the total surface area of the remaining solid.

(c) N non-overlapping holes are drilled lengthwise through the cylinder in (a).
If the total surface area of the remaining solid is *double* that of the cylinder in (a), find the value of N.

4. The diagram below shows two overlapping squares each of side-length 16 cm. AC is an axis of symmetry of the figure and M, N are the mid-points of AD and AB, respectively. Find the area of the small triangle PQC: give your answer correct to two decimal places.



5. (a) Explain briefly why the mean of an *odd* number of consecutive whole numbers is itself a whole number.

   (b) In a similar way, what can you say about the mean of an *even* number of consecutive whole numbers?

   (c) By using your answers to (a) and (b), find all sets of consecutive whole numbers which have a sum of 105.

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6. In this question, we will call a rectangle *special* if its area (in cm^2) is numerically equal to its perimeter (in cm).

(a) If a special rectangle has length x cm and width y cm, explain why the equation $xy = 2x + 2y$ holds.

(b) In (a), if $x = 2.5$ show that $y = 10$.

(c) For $x = 2.5, 3, 3.5, 4, 4.5, 6$, find the corresponding values of y for a special rectangle.

(d) Choosing sensible scales, plot a graph of y against x.

(e) (i) Which special rectangles have the additional property that x and y are whole numbers?

(ii) What happens to y if x is close to 2?

7. This question concerns the table of numbers below: the entry in Column D is the sum of the squares of the entries in Columns A, B, C.

	A	B	C	D
Row 1	1	2	2	9
Row 2	2	3	6	-
Row 3	3	4	12	-
Row 4	-	-	20	-
Row 5	-	-	-	-

(a) Copy this table and fill in the missing entries.

(b) Study the table carefully and write down formulae in terms of n for the values of A, B, C corresponding to Row n.

(c) Why are the entries in Column D all odd numbers?

(d) Study Columns C and D carefully and write down a formula for the entry for Column D in Row n.

(e) Use your answers to predict which whole number is the square root of $10000^2 + 10001^2 + 100010000^2$.